Analytical Study on the Angular Dependency of A_0-mode Lamb Wave in Transversely Isotropic Material

M S Rabbi
Department of Mechanical Engineering
Chittagong University of Engineering & Technology, Chattogram, Bangladesh

K Teramoto
Department of Advanced Technology Fusion
Saga University, Saga, Japan

H Ishibashi
Department of Advanced Technology Fusion
Saga University, Saga, Japan

Abstract- A_0-mode Lamb wave propagating characteristics in composites materials i.e. transversely isotropic material is studied in this research. Among a couple of types of Lamb wave and various modes in each type, A_0-mode attract a reasonable attention due to its lower phase velocity and it can be found at any operating frequency. The Mathematical model has been discussed to find out the major elastic constants involved in phase velocity dispersion in the media considered in this study. The model also reveals the impact of variation of fiber direction in the longitudinal and shear wave velocities. ANSYS workbench is used to develop the model whereas LS-DYNA explicit software has been used for the post-processing of the numerical analysis. The Lamb wave phase velocity direction skewing from the group velocity direction has been found at almost every angle and the wave propagation is dominant in the 0^o fiber direction of the lamina.

Keywords –Lamb wave, Transversely Isotropic material, ANSYS, LS-DYNA, Phase velocity.

I. INTRODUCTION

The Rayleigh-Lamb wave is generated in a plate with small thickness plate with stress-free surface. Usually, compression and shear waves have been combined to form Lamb wave whereas the particles are moving in elliptical orbit motion [1]. Lamb waves are the distinctive wave among all plate waves that are most widely encountered in NDT. Lamb waves can circulate throughout the thickness of the product perpendicular to the plate. Lamb wave propagating characteristics are solely depending on the density and the stiffness component of the considered material [2]. Lamb waves are formed at an angle of incident where the parallel portion of the propagation velocity in the origin is equivalent to that of in the material of the sample. Various types of particle motion are possible in Lamb waves, although Antisymmetrical (A) and Symmetrical (S) are governs commonly, illustrated in Fig. 1

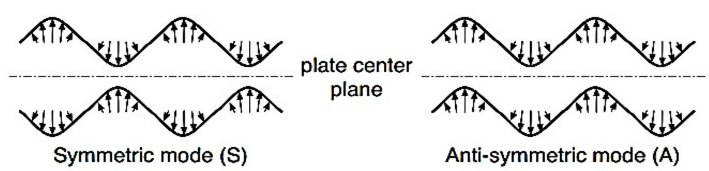

Figure 1. Out-of-plane direction of the particle in major Lamb wave modes[3]

The particles' composite movement is identical to the ground waves' elliptical orbits. S-waves move about the plate's median plane in a symmetrical fashion. This is sometimes referred to as the extensional mode as the wave "stretches and compresses" direction of wave motion [4]. In case of S-mode, the force is acting on the perpendicular of the plate. The A-mode wave is termed as flexural mode [5].

However, Lamb waves hold the multi-modal characteristics. Among various modes, zero order symmetric (S_0) and asymmetric (A_0) mode has been received much interest from the researchers because of their distinguished damage detection characteristics. Apart from this duo, fundamental order shear-horizontal (SH_0) mode has also been used for its non-dispersive properties in isotropic plate. A_0-mode Lamb wave draws more attention to the researchers' due to its slower phase velocity, shorter wavelength and it can be realized at higher spatial resolution. Thus, this kind of wave mode is more perfect to interrogate material for Non-destructive evaluation. Naturally, owing to the anisotropic properties of the material, the propagation behavior in such media is quite different than in isotropic plate for incident, scattered as well as transmitted cases. In this scenario, a combination of the longitudinal, shear-vertical and shear-horizontal waves have been exists. Therefore, it is impossible to solve the Lamb modes and the Love modes independently. The micro-mechanical behavior of anisotropic material depends on directions of the fiber, associated matrix composition, the thickness of the lamina and the arrangements in the direction of the plate thickness [6].

This research investigates the A_0-mode wave propagation directionality in anisotropic material and explained it for transversely isotropic material. Among two categories of TI material, horizontally TI material has been chosen where the horizontal axis considered as the plane of isotropy. Analytical model has been discussed thoroughly to investigate the polar dependency and numerical experiment has been conducted to proof the mathematical model.

The following articles are distributed as follows. Section II explains the Lamb wave dispersion characteristics. The material has been considered in this study along with the variable of longitudinal and shear phase velocities for such kind of material are discussed in section III. Section IV explains the mathematical model of the angular dependency criteria. Section V holds the numerical validation of the entire mathematical model. Finally, conclusion has been given in section VI.

II. DISPERSION CURVE OF LAMB WAVE

Considering a plate of thickness d, density ρ, longitudinal velocity C_L, and shear velocity C_T with a single sound source having angular frequency ω, the wave number k, the normal displacements can be expressed as

$$u_{sz}(x,y,t) = \left(a_{szL}\left(\exp(\mathrm{i}\beta_L) - \exp(-\mathrm{i}\beta_L)\right) + a_{szT}\left(\exp(\mathrm{i}\beta_T) - \exp(-\mathrm{i}\beta_T)\right)\right) \cdot \exp\left(\mathrm{i}\left(\omega t - k\sqrt{x^2+y^2}\right)\right), \quad (1)$$

$$u_{az}(x,y,t) = \left(a_{azL}\left(\exp(\mathrm{i}\beta_L) + \exp(-\mathrm{i}\beta_L)\right) + a_{azT}\left(\exp(\mathrm{i}\beta_T) + \exp(-\mathrm{i}\beta_T)\right)\right) \cdot \exp\left(\mathrm{i}\left(\omega t - k\sqrt{x^2+y^2}\right)\right), \quad (2)$$

The following out-of-plane displacements equation holds for the wave number:

$$\beta_L = \sqrt{\left(\frac{\omega}{C_L}\right)^2 - k^2}, \beta_T = \sqrt{\left(\frac{\omega}{C_T}\right)^2 - k^2}. \quad (3)$$

Under the zero-stress boundary conditions, the S and A Lamb waves satisfy the following Rayleigh-Lamb's equations respectively:

$$\frac{\tan(\beta_T d/2)}{\tan(\beta_L d/2)} = -\frac{4\beta_L\beta_T k^2}{(k^2-\beta_T^2)^2}, \qquad \frac{\tan(\beta_L d/2)}{\tan(\beta_T d/2)} = -\frac{4\beta_L\beta_T k^2}{(k^2-\beta_T^2)^2}. \quad (4)$$

Therefore, the Lamb wave dispersive curves propagating through an isotropic plate are plotted as Fig. 2.

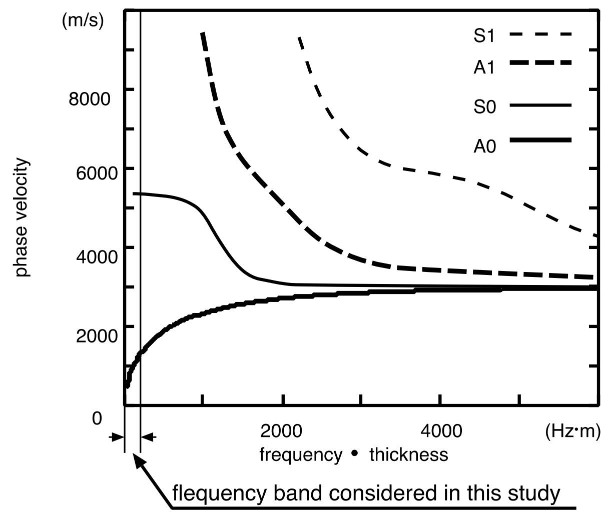

Figure 2. Dispersive characteristics for Lamb waves for isotropic plate with 1mm of thickness

The specific phase velocity can be matter of interest for particular application. The low-frequency A_0-mode Lamb wave is attractive for researchers as it can be observed in all operating frequencies. At every operating frequency, such mode is available and best suited for long-range NDT applications. As the composites are well established materials for various industrial equipments, study on A_0-mode Lamb wave propagation through such kind of material draws adequate interest to the researchers due to the perfect use as the testing media. The next section will explain the details of the material considered in this study.

III. Transversely Isotropic Material

Composites reinforced with unidirectional fibers can be regarded as the transversely isotropic material. The particle velocity vector can be stated as and the stress is in an elastic body and holds the following differential equations

$$\rho \frac{\partial v}{\partial t} = \nabla . T \tag{5}$$

$$\frac{\partial T}{\partial t} = R \nabla_S . v \tag{6}$$

Where, R is the stiffness matrix. Stiffness matrix of a typical medium can be expressed as follows:

$$\mathbb{R} = \begin{pmatrix} R_{1111} & R_{1122} & R_{1133} & R_{1123} & R_{1131} & R_{1112} & R_{1132} & R_{1113} & R_{1121} \\ R_{2211} & R_{2222} & R_{2233} & R_{2223} & R_{2231} & R_{2212} & R_{2232} & R_{2213} & R_{2221} \\ R_{3311} & R_{3322} & R_{3333} & R_{3323} & R_{3331} & R_{3312} & R_{3332} & R_{3313} & R_{3321} \\ R_{2311} & R_{2322} & R_{2333} & R_{2323} & R_{2331} & R_{2312} & R_{2332} & R_{2313} & R_{2321} \\ R_{3111} & R_{3122} & R_{3133} & R_{3123} & R_{3131} & R_{3112} & R_{3132} & R_{3113} & R_{3121} \\ R_{1211} & R_{1222} & R_{1233} & R_{1223} & R_{1231} & R_{1212} & R_{1232} & R_{1213} & R_{1221} \\ R_{3211} & R_{3222} & R_{3233} & R_{3223} & R_{3231} & R_{3212} & R_{3232} & R_{3213} & R_{3221} \\ R_{1311} & R_{1322} & R_{1333} & R_{1323} & R_{1331} & R_{1312} & R_{1332} & R_{1313} & R_{1321} \\ R_{2111} & R_{2122} & R_{2133} & R_{2123} & R_{2131} & R_{2112} & R_{2132} & R_{2113} & R_{2121} \end{pmatrix} \tag{7}$$

Using Voigt's notation and for transversely isotropic material where e_1 axis acts as the plane of isotropy, stiffness matrix can be expressed as

$$\mathbb{R} = \begin{pmatrix} R_{11} & R_{12} & R_{12} & 0 & 0 & 0 \\ R_{12} & R_{22} & R_{23} & 0 & 0 & 0 \\ R_{12} & R_{23} & R_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & R_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & R_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & R_{55} \end{pmatrix} \tag{8}$$

where R_{44} can be determined by $(R_{22}-R_{23})/2$.

In addition, the operator ∇ and   can be expressed as

$$\nabla = \begin{pmatrix} \frac{\partial}{\partial r_1} & 0 & 0 & 0 & \frac{\partial}{\partial r_3} & \frac{\partial}{\partial r_2} \\ 0 & \frac{\partial}{\partial r_2} & 0 & \frac{\partial}{\partial r_3} & 0 & \frac{\partial}{\partial r_1} \\ 0 & 0 & \frac{\partial}{\partial r_3} & \frac{\partial}{\partial r_2} & \frac{\partial}{\partial r_1} & 0 \end{pmatrix} \tag{9}$$

$$\nabla_S = \begin{pmatrix} \frac{\partial}{\partial r_1} & 0 & 0 \\ 0 & \frac{\partial}{\partial r_2} & 0 \\ 0 & 0 & \frac{\partial}{\partial r_3} \\ 0 & \frac{\partial}{\partial r_3} & \frac{\partial}{\partial r_2} \\ \frac{\partial}{\partial r_3} & 0 & \frac{\partial}{\partial r_1} \\ \frac{\partial}{\partial r_2} & \frac{\partial}{\partial r_1} & 0 \end{pmatrix} \tag{10}$$

is defined by

$$\boldsymbol{w}^{\mathrm{T}} = \begin{pmatrix} \boldsymbol{v}^{\mathrm{T}} & \boldsymbol{T}^{\mathrm{T}} \end{pmatrix} \tag{11}$$

$$H = \begin{pmatrix} \mu & 0 & 0 & \\ 0 & \mu & 0 & 0 \\ 0 & 0 & \mu & \\ & 0 & & C \end{pmatrix} \tag{12}$$

From the equation 5 and 6, it can be write that

$$\frac{\partial \boldsymbol{w}}{\partial t} = H \begin{pmatrix} 0 & \nabla. \\ \nabla_S & 0 \end{pmatrix} \boldsymbol{w} \tag{13}$$

This is expressed in the form of the sum of partial differentials in the e_1, e_2, and e_3 directions, then

$$\frac{\partial \boldsymbol{w}}{\partial t} = H_1 \frac{\partial \boldsymbol{w}}{\partial r_1} + H_2 \frac{\partial \boldsymbol{w}}{\partial r_2} + H_3 \frac{\partial \boldsymbol{w}}{\partial r_3} \tag{14}$$

The differential equation is given by

$$H_1 = \begin{pmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{pmatrix}$$

$$H_{11} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad H_{12} = \begin{pmatrix} 1/\rho & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad H_{13} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1/\rho \\ 0 & 1/\rho & 0 \end{pmatrix}$$

$$H_{21} = \begin{pmatrix} R_{11} & 0 & 0 \\ R_{12} & 0 & 0 \\ R_{12} & 0 & 0 \end{pmatrix} \quad H_{22} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad H_{23} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{15}$$

$$H_{31} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & R_{55} \\ 0 & R_{55} & 0 \end{pmatrix} \quad H_{32} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad H_{33} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

And the diagonal matrix D_1 of eigen values, the corresponding eigenvectors consisting of L_1, and the following relation can be written

$$H_1 = L_1 D_1 L_1^{-1} \tag{16}$$

The matrix consists of 9×9 = 81 components. The first six diagonal components can be evaluated as the following whereas rest are become zero.

$$D_{11} = D_{22} = \pm\sqrt{\frac{R_{11}}{\rho}} \text{ and } D_{33} = D_{44} = D_{55} = D_{66} = \pm\sqrt{\frac{R_{55}}{\rho}} \tag{17}$$

The longitudinal phase velocity in e_1 direction is v_{11}, the shear wave velocity from e_1 direction to e_2 direction is v_{12}, and the shear wave velocity from e_1 direction to e_3 direction can be termed as v_{13}. These velocities can be expressed as

$$v_{11} = \sqrt{\frac{R_{11}}{\rho}}, v_{12} = \sqrt{\frac{R_{55}}{\rho}}, v_{12} = \sqrt{\frac{R_{55}}{\rho}} \tag{18}$$

Similarly, the phase velocity travelling in the e_2 direction is termed as v_{22}. Shear wave velocity associated with the particle displacement from e_2 direction to e_3 direction is v_{23}. The phase velocity v_{21} denotes that the particle displacement in the e_1 direction advances from e_2 direction.

$$v_{22} = \sqrt{\frac{R_{22}}{\rho}}, v_{23} = \sqrt{\frac{(R_{22} - R_{23})}{\rho}}, v_{21} = \sqrt{\frac{R_{55}}{\rho}} \tag{19}$$

The fiber directional anisotropic behavior is explained in the next section.

IV. Angular Depenedence of A_0-mode Lamb Wave

Theoratically, the wave is propagating along the way rotated by in the e_3 direction with respect to the e_1 axis. The variation in the phase velocities depends upon the rotation of the stiffness tensor and calculated by applying the following transformation [7, 8]:

$$R_{ijkl}(\theta) = \sum_{m=1}^{3}\sum_{n=1}^{3}\sum_{r=1}^{3}\sum_{s=1}^{3} p_{mi}p_{nj}p_{rk}p_{sl}R_{mnrs} \tag{20}$$

The rotation matrix can be expressed accordingly as:

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{21}$$

The transformed stiffness matrix *R( )* is the orthotropic material given by

$$R(\theta) = \begin{pmatrix} R_{11}(\theta) & R_{12}(\theta) & R_{13}(\theta) & 0 & 0 & 0 \\ R_{21}(\theta) & R_{22}(\theta) & R_{23}(\theta) & 0 & 0 & 0 \\ R_{31}(\theta) & R_{32}(\theta) & R_{33}(\theta) & 0 & 0 & 0 \\ 0 & 0 & 0 & R_{44}(\theta) & 0 & 0 \\ 0 & 0 & 0 & 0 & R_{55}(\theta) & 0 \\ 0 & 0 & 0 & 0 & 0 & R(\theta) \end{pmatrix} \tag{22}$$

where each components can be obtained as:

$$R_{11}(\theta) = R_{11}\cos^4\theta + R_{22}\sin^4\theta + (2R_{11} + 4R_{55})\cos^2\theta\sin^2\theta$$
$$R_{12}(\theta) = R_{12}(\cos^4\theta + \sin^4\theta) + (R_{11} + R_{22} - 4R_{55})\cos^2\theta\sin^2\theta$$
$$R_{13}(\theta) = R_{12}\cos^2\theta + R_{23}\sin^2\theta$$
$$R_{22}(\theta) = R_{22}\cos^4\theta + R_{11}\sin^4\theta + (2R_{12} + 4R_{55})\cos^2\theta$$
$$R_{23}(\theta) = R_{12}\sin^2\theta + R_{23}\cos^2\theta \qquad (23)$$
$$R_{33}(\theta) = R_{22}$$
$$R_{44}(\theta) = \frac{R_{22} - R_{23}}{2}\cos^2\theta + R_{55}\sin^2\theta$$
$$R_{55}(\theta) = \frac{R_{22} - R_{23}}{2}\sin^2\theta + R_{55}\cos^2\theta$$
$$R_{66}(\theta) = (R_{11} + R_{22} - 2R_{12})\cos^2\theta\sin^2\theta + R_{55}(\cos^2\theta - \sin^2\theta)^2$$

Here,

$$e_1(\theta) = e_1\cos\phi + e_2\sin\phi, e_2(\theta) = -e_1\cos\theta + e_2\sin\theta \qquad (24)$$

The longitudinal phase velocities in the $e_1( )$ direction and shear wave phase velocities from $e_1( )$ direction to $e_2( )$-way and $e_3( )$-way can be expressed as

$$v_{11}(\phi) = \sqrt{\frac{R_{11}(\theta)}{\rho}}, v_{12}(\theta) = \sqrt{\frac{R_{66}(\theta)}{\rho}}, v_{13}(\theta) = \sqrt{\frac{R_{55}(\theta)}{\rho}} \qquad (25)$$

The TI material properties considered as the following:

- Fiber orientation: e_1
- Mass density ρ : 1500 kg/m^3
- R_{11} = 96.0 (GPa)
- R_{12} = 3.60 (GPa)
- R_{22} = 9.60 (GPa)
- R_{23} = 7.01 (GPa)
- R_{55} = 3.30 (GPa)

Calculated phase velocity disersion curves based on the values considering for A_0-mode Lamb wave propagating in this elastic body are illustrated in Fig. 3 (0^{o}fiber direction as e_1 direction and 90^{o}fiber direction as e_3 direction). It's a clear variation in the phase velocities have been found and fiber direction enhances the wave propagation characteristics. It is noteworthy that for the specific combination of frequency and material thickness, the phase velocities in any fiber direction other than this two, would be within this maximum (found in 0^o direction) and minimum (found in 90^o direction) values.

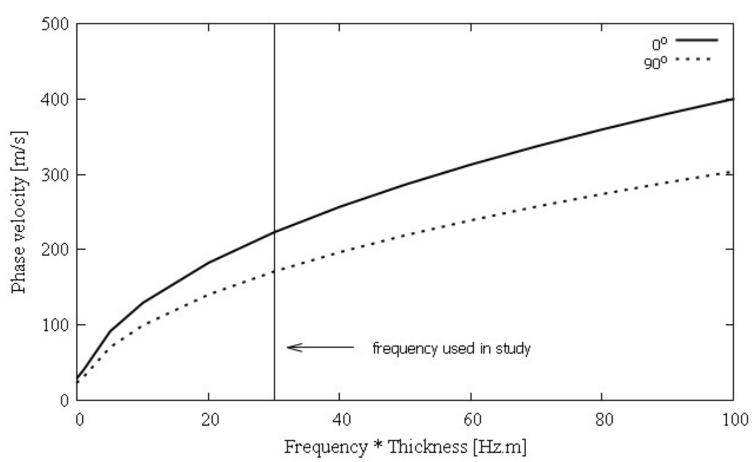

Figure 3. Theoretically calculated phase velociyt dispersion curves for A_0-mode Lamb wave propagating along the fiber axis (solid plot) and the perpendicular direction (dots plot) in TI material of 1.00 mm in thickness.

IV. NUMERICAL STUDY

The geometrical model, meshing has been created by ANSYS Workbench. Explicit dynamics (LS-DYNA Export) analysis has been used to perform the post processing analysis. Properties can be stored in libraries of materials. A rectangular plate of having 75 mm × 30 mm × 1 mm dimension has been drawn in the ANSYS Design Modeler Window (depicted in Fig. 4).

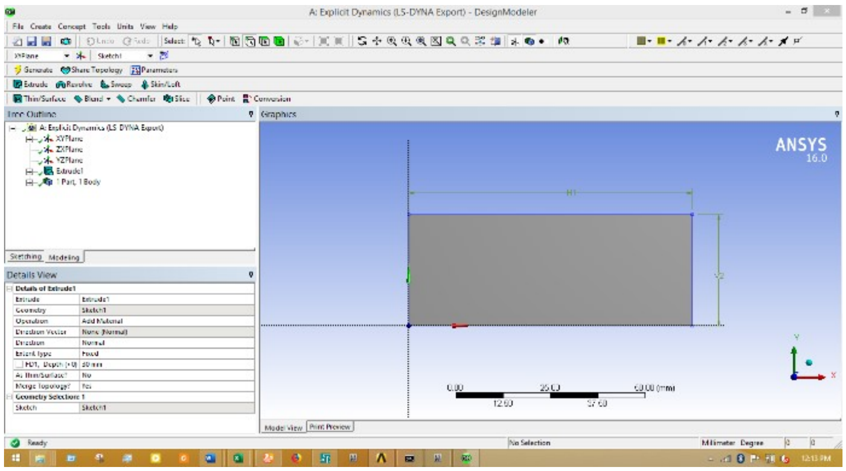

Figure 4. Rectangular plate model in ANSYS design modeler window

Automatic meshing method has been used in this study to conduct the simulation. The element size has been set to 0.1 mm and 0.25 mm in thickness and length directions, respectively to keep maintain the proper aspect ratio for getting better result. The mesh was generated on each of the designs with the proper enclosure. For applying the load, a single cycle sinusoidal tone burst has been applied at the edge of one end of rectangular plate and other surfaces are remained stress-free as this condition is too crucial for generating Lamb wave. 30 kHz operating frequency has been chosen. Initial time step chosen was set at 1×10^{-7} sec and time interval between outputs was set at 1×10^{-4} sec. LS-DYNA Program Manager is the LS-DYNA or LS-PrePost PC / Windows GUI frontier. LS-DYNA is equipped with both single precision and double precision executables. In the simulating result, Lamb wave propagation for 30 kHz can be found by observing the normal displacement of the particles and illustrated in Fig. 5.

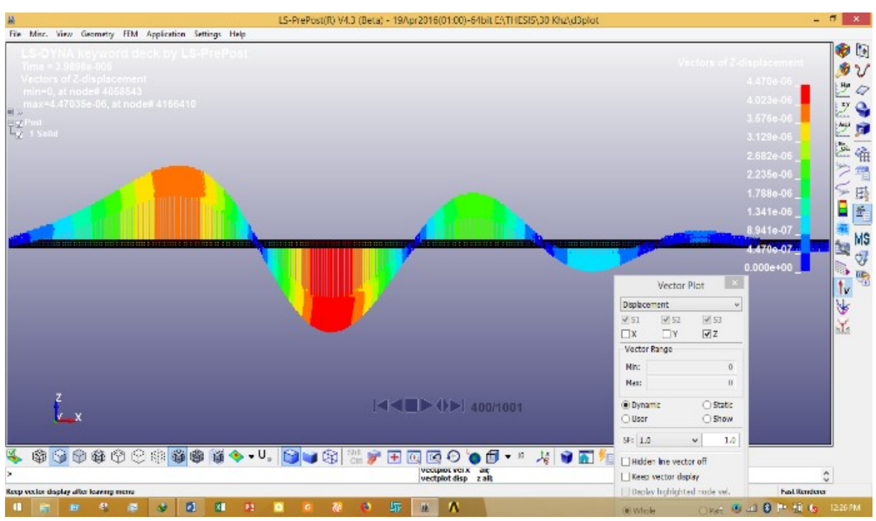

Figure 5. Lamb wave propagation in TI material

At any particular location, normal motion at the mid-plane of the model ensures the zero-mode anti-symmetric Lamb wave signal providing that the S_0 and SH_0-mode have zero normal motion at that plane [9]. Following identical way, a sinusoidal wave load has been applied to a point source and observes its propagation along all directions. To trace the angular dependency in various directions, in every 30^o angular position, phase velocities have been calculated. To do so, several measurement points were taken along various directions. It is confirmed that excitation signal wavelength is more than double than the measurement point distance. The distance considered in such way to valid the far-field wave field solution [10]. By making the average of all measurement points, phase velocity of the propagation is determined. Fig.6 illustrates the angular dependency of the phase velocity of fundamental order anti-symmetric (A_0)-mode phase velocity from the theoretical calculation and FE simulation which resembles a good agreement between those.

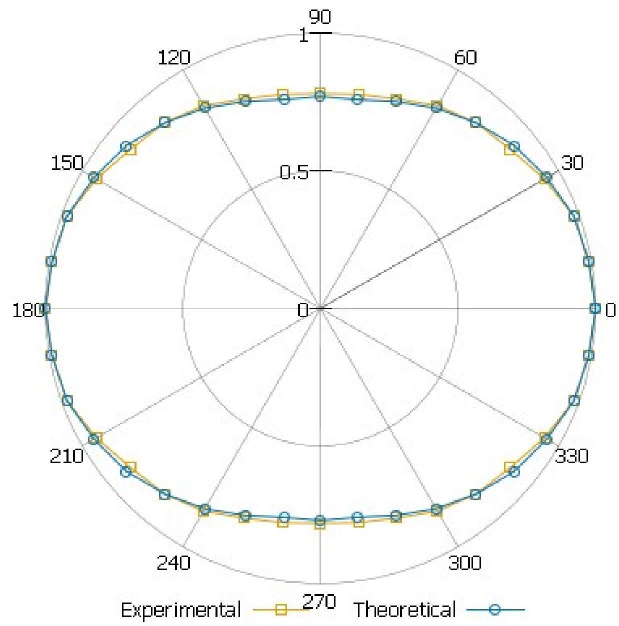

Figure 6. Angular dependence of phase velocity from theoretical calculation and FE simulations for horizontally TI material at 30 kHz frequency

Each value of the phase velocity is normalized by its maximum value. Both lines indicate that the direction of phase velocity is skewed from the group velocity direction at almost every angle and the wave is propagating with more velocity in the fiber direction of the layer. Thus, it can be said that the excited signal has the capability to follow the theoretical phase velocity of A_0-mode Lamb wave.

V.CONCLUSION

Lamb wave is a specific type of plate wave, have been widely used to identify the defect in various materials. Composite materials have been used for a few decades in various engineering fields. To develop the damage detecting algorithm in composites using most effective mode i.e. A_0-mode of Lamb wave, the prerequisite activity is to measure the wave propagating properties in anisotropic composite materials. Transversely isotropic material has been used as lamina in every cross-ply or quasi-isotropic laminates. This study has been done to find out the directivity pattern of A_0-mode Lamb wave propagation in transversely isotropic material. To proof the mathematical model, computational study has been conducted. It is found good agreement between this two study. It is found that, Lamb wave propagates with higher phase velocity in the fiber direction. An elliptical profile has been found while considering all directions. Study on the Lamb wave propagation through cross-ply laminates with various combinations of lamina could be a scope of future study.

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